1 Flipping game

1.1 Description

Suppose that \((\Sigma, E, R)\) is the following rewrite theory:

\[
\Sigma: \quad \text{sort \ State}
\]

operation \(\emptyset : \rightarrow \text{State}\)

operation \(- : \text{State} \times \text{State} \rightarrow \text{State}\)

operation \(0 : \rightarrow \text{State}\)

operation \(1 : \rightarrow \text{State}\)

\[
E: \quad \text{equation} \quad (\forall S: \text{State}) \emptyset S = S
\]

\[
\text{equation} \quad (\forall S_1, S_2: \text{State}) S_1 S_2 = S_2 S_1
\]

\[
\text{equation} \quad (\forall S_1, S_2, S_3: \text{State}) (S_1 S_2) S_3 = S_1 (S_2 S_3)
\]

\[
R: \quad \text{rule} \quad r_1 : 0 \Rightarrow 1
\]

\[
\text{rule} \quad r_2 : 1 \Rightarrow 0
\]

The two equations state the associativity and commutativity of the binary “\(-\)” operator, thus making it a multi-set operator, and the two rules flip the two constants 0 and 1.

1.2 Requirements

1. Using exhaustive search, show that it is possible to start with the state of zeros 0 0 0 0 0 and reach a state of just ones.

2. Using the model checker, show that it is not the case that whenever one reaches the state of zeros then one will eventually reach a state of ones.

2 Missionaries and Cannibals

2.1 Description (from Wikipedia)

Three missionaries and three cannibals stand on the bank of a river that they wish to cross. There is a boat available which can ferry up to two people
across. The goal is to find a schedule for ferrying all the cannibals and all the missionaries safely across the river. The constraint is that, if at any point the cannibals outnumber the missionaries on either bank, the cannibals will eat the missionaries. Note that the boat cannot cross the river by itself with no people on board.

2.2 Requirements

1. Define the above problem as a rewrite theory in Maude. Note that you should only define the moves, not specify a solution.

2. Use Maude to find a solution to the problem as specified above.

3 Dining Philosophers

3.1 Description (from Wikipedia)

The dining philosophers problem is summarized as five philosophers sitting at a table doing one of two things - eating or thinking. While eating, they are not thinking, and while thinking, they are not eating. The five philosophers sit at a circular table with a large bowl of rice in the center. A chopstick is placed in between each philosopher, and as such, each philosopher has one chopstick to his or her left and one chopstick to his or her right. In order for a philosopher to eat, the philosopher must have two chopsticks. In the case of the dining philosopher, the philosopher can only use the chopstick on his or her left or right.

3.1.1 Deadlock

Deadlock is when none of the actors can proceed because it is waiting for resources held by others.

3.1.2 Starvation

Starvation is when there exists an execution in which a process, although is is enabled to perform an action infinitely often, it does not actually perform it.

3.2 Requirements

1. Define the dining philosophers problem in Maude and prove that it “suffers” from both deadlock and starvation.

2. Propose a fix to the deadlock problem and prove (using search) that it is indeed a fix. Also check if the fix you proposed solves the starvation problem as well.

Hint: One simple fix is to have the philosophers on odd positions take the left chopstick first, while those on even positions take the right one first.