K and Matching Logic

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Question

... could it be that, after 40 years of program verification, we still lack the right semantically grounded program verification foundation?

\[
\begin{align*}
\{ \pi_{\text{pre}} \} & \quad \text{code} & \{ \pi_{\text{post}} \}
\end{align*}
\]

Hoare logic
Current State-of-the-Art in Program Analysis and Verification

Consider some programming language, L

- **Formal semantics of L?**
  - Typically skipped: considered expensive and useless

- **Model checkers for L**
  - Based on some adhoc encodings/models of L

- **Program verifiers for L**
  - Based on some other adhoc encodings/models of L

- **Runtime verifiers for L**
  - Based on yet another adhoc encodings/models of L

- ...
Example of C Program

• What should the following program evaluate to?

```c
int main(void) {
    int x = 0;
    return (x = 1) + (x = 2);
}
```

• According to the C “standard”, it is **undefined**

• GCC4, MSVC: it returns **4**
  GCC3, ICC, Clang: it returns **3**

By April 2011, both Frama-C (with its Jessie verification plugin) and Havoc "prove" it returns **4**
A Formal Semantics Manifesto

• Programming languages must have formal semantics! (period)
  – And analysis/verification tools should build on them at best, or should formally relate to them at worst
    • Otherwise they are adhoc and likely wrong

• Informal manuals are not sufficient
  – Manuals typically have a formal syntax of the language (in an appendix)
  – Why not a formal semantics appendix as well?
Motivation and Goal

• We are facing a semantic chaos
  – Axiomatic, denotational, operational, etc.

• Why so many semantic styles?
  – Since none of them is ideal, they have limitations

• We want a powerful, unified foundation for language design, semantics and verification
  – One semantic approach to serve all the purposes!
  – To work with realistic languages (C, Java, etc.)
Minimal Requirements for an Ideal Language Semantic Framework

• Should be expressive
  – Substitution or environment-based definitions, abrupt control changes (callcc), concurrency, etc.

• Should be executable
  – So we can test it and use it in tools (symb. exec.)

• Should be modular (thus scale)
  – So each feature is defined once and for all

• Should serve as a basis for program reasoning
  – So we can also prove programs correct with it
- Conventional Semantic Approaches -
  - Advantages and Limitations –

Chronologically
- 1969: Floyd-Hoare Logic -

- Basis for program verification
- Not executable, and thus, hard to test
  - Semantic errors found by proving wrong properties
  - Soundness rarely or never proved in practice
- Not very expressive
  - Often requires heavy program transformations (e.g., to eliminate side effects, pointers, exceptions, etc.), to reduce languages to cores which can be given an axiomatic semantics
  - Structural program properties (e.g., about heap, stacks, input/output, etc.) hard to state; need special logic support
  - Structural framing (e.g., heap framing) hard to deal with
- Implementations of Floyd-Hoare verifiers for real languages still an art, who few master
Reasonable trade-offs. “Compiles” programs into mathematical objects, so it can be *in principle*:

- **Expressive**, provided enough/appropriate mathematical domains available
- **Executable**: we can execute/approximate fixed-points
  - Although factorial(5) crashes Papaspyrou’s C semantics
- **Modular**, provided one uses advanced features
  - Monads, continuations, resumptions
- **Basis for program verification**
  - Program = least fixed point, so we can use induction
- **Hard to use and understand; requires expert knowledge; no overwhelming evidence it is practical for verification**
- 1981: Operational Semantics -

Quite intuitive, easy to understand and define. Requires minimal training and it scales.

- **Executable**, by its very nature
  - Although Norish’s C semantics not executable, evaluation contexts are inefficient, CHAM has no machine support, etc.

- **Expressive** ... in principle
  - Although hard to use both evaluation contexts (for call/CC, longjumps,...) and environment-store (for pointers, threads,...)

- **Modular**, when one uses
  - MSOS ideas for dealing with configuration changes, evaluation contexts ideas for control, CHAM ideas for concurrency, etc.

- Considered “too low level”, inappropriate for verification
Towards a Better Semantic Approach

• First, we want a semantic framework which, at the same time and uniformly well, is
  – Expressive, Executable, Modular, and
  – Suitable for program reasoning

• Second, we want to develop supporting tools for
  – Defining formal language semantics, and
  – Using the semantics for program verification
    • Put an end to having both operational and axiomatic or other semantics to languages. No more semantics equivalence proofs to be done and maintained!
Starting Point: Rewriting Logic

Meseguer (late 80s, early 90s)

• Expressive
  – Any logic can be represented in RL (it is reflective)

• Executable
  – Quite efficiently; Maude often outperforms SML

• Modular
  – Allows rules to only “match” what they need

• Can serve as a basis for program reasoning
  – Admits initial model semantics, so it is amenable for inductive or fixed-point proofs
Rewriting Logic Semantics Project

• Project started jointly with Meseguer in 2003-4
• Idea: Define the semantics of a programming language as a rewrite theory (set of rules)
• Showed that most executable semantics approaches can be framed as rewrite logic semantics (Modular/SmallStep/BigStep SOS, evaluation contexts, continuation-based, etc.)
  – But they still had their inherent limitations
• Appropriate techniques/methodologies needed
The K Framework

k-framework.org

• A tool-supported rewrite-based framework for defining programming language semantics
• Inspired from rewriting logic
• Used regularly in teaching
• Main ideas:
  – Represent program configurations as a potentially nested structure of cells (like in the CHAM)
  – Flatten syntax into special computational structures (like in refocusing for evaluation contexts)
  – Define the semantics of each language construct by semantic rules (a small number, typically 1 or 2)
Complete K Definition of KernelC
Syntax declared using annotated BNF

SYNTAX

\[ \text{Exp} ::= \text{Exp} \mid \text{Exp} = \text{Exp} \ [\text{strict}(2)] \]
Complete K Definition of KernelC

Configuration given as a nested cell structure. Leaves can be sets, multisets, lists, maps, or syntax.
Complete K Definition of KernelC

Semantic rules given contextually

<k> X = V => V ...</k>
<env>... X |-> (_ => V) ...</env>
K Semantics are Useful

• Executable, help language designers
• Make teaching PL concepts hands-on and fun
• Currently compiled into
  – Maude, for execution, debugging, model checking
  – Latex, for human inspection and understanding
• Plans to be compiled to
  – OCAML, for fast execution
  – COQ, for meta-property verification
Medium-Size K Definition

- See the semantics of SIMPLE on the “K and Matching Logic” page
K Scales

Besides smaller and paradigmatic teaching languages, several larger languages were defined
• Scheme : by Pat Meredith
• Java 1.4 : by Feng Chen
• Verilog : by Pat Meredith and Mike Katelman
• C : by Chucky Ellison
etc.
The K Configuration of C

Heap

75 Cells!
Statistics for the C definition

• Total number of rules: ~1200

• Has been tested on thousands of C programs (several benchmarks, including the gcc torture test, code from the obfuscated C competition, etc.)
  – Passed 99.2% so far!
  – GCC 4.1.2 passes 99%, ICC 99.4%, Clang 98.3 (no opt.)

• The most complete formal C semantics

• Took more than 18 months to define ...
  – Wouldn’t it be uneconomical to redefine it in each tool?
Matching Logic Verification

= Rewriting (language semantics) + [FOL] (configuration reasoning) + Proof Rules (behavior reasoning)
Matching Logic

• A logic for reasoning about configurations
  • Formulae
    – [FOL] over configurations, called patterns
    – Configurations are allowed to contain variables
  • Models
    – Ground configurations
• Satisfaction
  – Matching for configurations, plus [FOL] for the rest
Examples of Patterns

• x points to sequence A with $|A| > 1$, and the reversed sequence $\text{rev}(A)$ has been output.

• `untrusted()` can only be called from `trusted()`.
More Formally: Configurations

• For concreteness, assume configurations having the following syntax:

\[
\langle \langle \ldots \rangle_k \langle \ldots \rangle_{env} \langle \ldots \rangle_{heap} \langle \ldots \rangle_{in} \langle \ldots \rangle_{out} \ldots \rangle_{cfg}
\]

(matching logic works with any configurations)

• Examples of concrete (ground) configurations:

\[
\begin{align*}
&\langle \langle x=^*y ; y=x ; \ldots \rangle_k \langle x \mapsto 7, y \mapsto 3, \ldots \rangle_{env} \langle 3 \mapsto 5 \rangle_{heap} \ldots \rangle_{cfg} \\
&\langle \langle x \mapsto 3 \rangle_{env} \langle 3 \mapsto 5, 2 \mapsto 7 \rangle_{heap} \langle 1, 2, 3, \ldots \rangle_{in} \langle \ldots, 7, 8, 9 \rangle_{out} \ldots \rangle_{cfg}
\end{align*}
\]
More Formally: Patterns

• Concrete configurations are already patterns, but very simple ones, ground
• Example of more complex pattern

\[ \exists c : \text{Cells}, e : \text{Env}, p : \text{Nat}, i : \text{Int}, \sigma : \text{Heap} \]
\[ \langle \langle x \mapsto p, e \rangle_{\text{env}}, \langle p \mapsto i, \sigma \rangle_{\text{heap}}, c \rangle_{\text{cfg}} \land i > 0 \land p \neq i \]

• Thus, patterns generalize both terms and [FOL]
• Models: concrete configurations + valuations
• Satisfaction: matching for patterns, [FOL] for rest
More Formally: Reasoning

• We can now prove (using FOL reasoning) properties about configurations, such as

\[ \forall c : \text{Cell}, \ e : \text{Env}, \ p : \text{Nat} \]
\[ \langle \langle x \mapsto p, \ e \rangle_{\text{env}} \langle p \mapsto 9 \rangle_{\text{heap}} \rangle_{\text{cfg}} \land p > 10 \]
\[ \rightarrow \exists i : \text{Int}, \ \sigma : \text{Heap} \]
\[ \langle \langle x \mapsto p, \ e \rangle_{\text{env}} \langle p \mapsto i, \ \sigma \rangle_{\text{heap}} \rangle_{\text{cfg}} \land i > 0 \land p \neq i \]
Matching Logic vs. Separation Logic

• Matching logic achieves separation through matching at the structural (term) level, not through special logical connectives (*)

• Matching logic realizes separation at all levels of the configuration, not only in the heap
  – the heap was only 1 out of the 75 cells in C’s def.

• Matching logic can stay within FOL, while separation logic needs to extend FOL
  – Thus, we can use the existing SMT provers, etc.
Matching Logic as a Program Logic

• Hoare style - not recommended

\{ \pi_{\text{pre}} \} \ \text{code} \ \{ \pi_{\text{post}} \}

  – One has to redefine the PL semantics – impractical

• Rewriting (or K) style – recommended

  \text{left[code]} \rightarrow \text{right}

  – One can reuse existing K semantics – very good
Example – Swapping Values

- What is the K semantics of the swap function?
- Let $ be its body

\[
\begin{align*}
\text{void swap}(\text{int }*x, \text{int }*y) \\
\{ \\
\quad \text{int } t; \\
\quad t=*x; \\
\quad *x=*y; \\
\quad *y=t; \\
\}
\end{align*}
\]

\[
\begin{align*}
\text{rule } \langle k \rangle \ $ & \rightarrow \text{return}; \quad \ldots \langle k \rangle \\
\langle \text{heap} \rangle & \ldots \\
& \quad x \rightarrow (a\rightarrow b), \\
& \quad y \rightarrow (b\rightarrow a) \\
& \quad \ldots \langle \text{heap} \rangle
\end{align*}
\]

\[
\begin{align*}
\text{if } x = y \\
\text{rule } \langle k \rangle \ $ & \rightarrow \text{return}; \quad \ldots \langle k \rangle \\
\langle \text{heap} \rangle & \ldots \\
& \quad x \rightarrow a \quad \ldots \langle \text{heap} \rangle
\end{align*}
\]
Example – Reversing a list

```c
struct listNode* reverse(struct listNode *x)
{
    struct listNode *p;
    struct listNode *y;
    p = 0;
    while(x) {
        y = x->next;
        x->next = p;
        p = x;
        x = y;
    } return p;
}
```

- What is the K semantics of the reverse function?
- Let $\$ be its body

```
rule <k> $ \Rightarrow \text{return } p; </k>
<heap>... \text{list}(x,A) \Rightarrow \text{list}(p,\text{rev}(A)) ...</heap>
```
Partial Correctness

• We have two rewrite relations on configurations
  \[ \rightarrow \] given by the language K semantics; safe
  \[ \rightarrow \] given by specifications; unsafe, has to be proved

• Idea (simplified for deterministic languages):
  – Pick \( \text{left} \rightarrow \text{right} \). Show that always \( \text{left} \rightarrow (\rightarrow \cup \rightarrow)^* \text{right} \)
    modulo matching logic reasoning (between rewrite steps)

• Theorem (soundness):
  – If \( \text{left} \rightarrow \text{right} \) and “config matches left” such that config
    has a normal form for \( \rightarrow \), then “nf(config) matches right”
More Formally: Matching Logic Rewriting

• Matching logic rewrite rules are rewrite rules over matching logic formulae: \( \varphi \Rightarrow \varphi' \)

• Since patterns generalize terms, matching logic rewriting captures term rewriting

• Moreover, deals naturally with side conditions: rewrite rules of the form

\[
l \Rightarrow r \text{ if } b
\]

are captured as matching logic rules of the form

\[
l \land b \Rightarrow r
\]
More Formally: Proof System I

- Rules of operational nature

**Reflexivity**

\[
\begin{align*}
\varphi & \Rightarrow \varphi \\
\mathcal{A} & \vdash \varphi \Rightarrow \varphi
\end{align*}
\]

**Axiom**

\[
\begin{align*}
\varphi & \Rightarrow \varphi' \\
\mathcal{A} & \vdash \varphi \Rightarrow \varphi'
\end{align*}
\]
More Formally: Proof System II

Substitution

\[ \mathcal{A} \vdash \varphi \Rightarrow \varphi' \quad \theta : \text{Var} \rightarrow \mathcal{T}_\Sigma(\text{Var}) \]

\[ \mathcal{A} \vdash \theta(\varphi) \Rightarrow \theta(\varphi') \]

Transitivity

\[ \mathcal{A} \vdash \varphi_1 \Rightarrow \varphi_2 \quad \mathcal{A} \vdash \varphi_2 \Rightarrow \varphi_3 \]

\[ \mathcal{A} \vdash \varphi_1 \Rightarrow \varphi_3 \]
More Formally: Proof System III

• Rules of deductive nature

Case analysis

\[
\frac{\mathcal{A} \vdash \varphi_1 \Rightarrow \varphi \quad \mathcal{A} \vdash \varphi_2 \Rightarrow \varphi}{\mathcal{A} \vdash \varphi_1 \lor \varphi_2 \Rightarrow \varphi}
\]

Logic framing

\[
\frac{\mathcal{A} \vdash \varphi \Rightarrow \varphi' \quad \psi \text{ is a FOL}_- \text{ formula}}{\mathcal{A} \vdash \varphi \land \psi \Rightarrow \varphi' \land \psi}
\]
More Formally: Proof System IV

**Consequence**

\[
\begin{align*}
& \models \varphi_1 \rightarrow \varphi'_1 \\
& \mathcal{A} \vdash \varphi'_1 \Rightarrow \varphi'_2 \\
& \models \varphi'_2 \rightarrow \varphi_2 \\
\hline
& \mathcal{A} \vdash \varphi_1 \Rightarrow \varphi_2
\end{align*}
\]

**Abstraction**

\[
\begin{align*}
& \mathcal{A} \vdash \varphi \Rightarrow \varphi' \\
& X \cap \text{FreeVars}(\varphi') = \emptyset \\
\hline
& \mathcal{A} \vdash \exists X \varphi \Rightarrow \varphi'
\end{align*}
\]
More Formally: Proof System V

- Main proof rule of matching logic rewriting

\[
\begin{align*}
A \vdash \varphi & \Rightarrow^+ \varphi'' \\
A \cup \{\varphi \Rightarrow \varphi'\} & \vdash \varphi'' \Rightarrow \varphi'
\end{align*}
\]

\[
A \vdash \varphi \Rightarrow \varphi'
\]
Fact

• Matching logic generalizes both operational semantics and axiomatic semantics
  – Operational semantics by means of capturing term rewriting as discussed above
  – Axiomatic semantics by noticing that Hoare triples are particular pattern rewrites:

\[
\text{HL2ML}({\{\psi\}} \ s \ {\{\psi'\}}) = \langle s, \sigma_Z \rangle \land \sigma_Z(\psi) \Rightarrow \exists Z(\langle \text{skip}, \sigma_Z \rangle \land \sigma_Z(\psi'))
\]
Theorem

• Any operational behavior can also be derived using matching logic reasoning

• For any Hoare triple $\{\psi\} s \{\psi'\}$ derived with axiomatic semantics, the corresponding matching logic rule $\text{HL2ML} (\{\psi\} s \{\psi'\})$ can be derived with the matching logic proof system
  – Proof is constructive, not existential

• Partial correctness
  – Holds for ALL languages
MatchC

- A Matching Logic Verifier for (a fragment of) C
- Uses the K semantics of the C fragment *unchanged*
- Has verified a series of challenging programs
  - Undefiness, typical Hoare-like programs, heap programs (lists, trees, stacks, queues, graphs), sortings, AVL trees, Schorr-Waite graph marking
Conclusions

• K (semantics) and Matching Logic (verification)
• Formal semantics is useful and practical!
• One can use an executable semantics of a language *as is* also for program verification
  – As opposed to redefining it as a Hoare logic
• Giving a formal semantics is not necessarily painful, it can be fun if one uses the right tools