

Rule systems for run-time monitoring: from EAGLE to RULER

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Abstract. EAGLE was introduced as a general purpose rule-based temporal logic for specifying run-time monitors. A novel and relatively efficient interpretative trace-checking scheme via stepwise transformation of an EAGLE monitoring formula was defined and implemented. However, application in real-world examples has shown efficiency weaknesses, especially those associated with large-scale symbolic formula manipulation. In this paper, after briefly reviewing EAGLE, we introduce RULER, a primitive conditional rule-based system, which we claim can be more efficiently implemented for run-time checking, and into which one can compile various temporal logics used for run-time verification.

1 Introduction

A plethora of logics have been used for the specification of behavioural system properties that can be dynamically checked either on-line throughout an execution of the system or off-line over an execution trace of the system. Some form of linear-time temporal logic usually forms the basis for the specification logic. This large variety of logics prompted the search for a small and general framework for defining monitoring logics, which would be powerful enough to capture most of the existing logics, thus supporting future and past time logics, interval logics, extended regular expressions, state machines, real-time and data constraints, and stochastic behaviour. The framework should support the definition of new logics easily and the monitoring of programs with their complex program states. EAGLE [3] was the result.

The EAGLE logic is a restricted first order, fixed-point, linear-time temporal logic with chop (concatenation) over *finite* traces. As such, the logic is highly expressive and, not surprisingly, EAGLE's satisfiability (validity) problem is undecidable; checking satisfiability in a given model, however, is decidable and that is what's required for run-time verification. The syntax and semantics of EAGLE is succinct. There are four primitive temporal operators: \bigcirc — next, \odot — previously, \cdot — concatenation, and $;$ — chop (overlapping concatenation, or sequential composition). Temporal equations can be used to define schema for temporal formulae, where the temporal predicates may be parameterized by data as well as by EAGLE formulas. The usual boolean logical connectives exist.

For example, the linear-time \square , \mathcal{U} and \mathcal{S} temporal operators can be introduced through the following equational definitions.

$$\begin{aligned} \mathbf{max} \text{ Always}(\mathbf{Form} F) &= F \wedge \bigcirc \text{Always}(F) \\ \mathbf{min} \text{ Sometime}(\mathbf{Form} F) &= F \vee \bigcirc \text{Sometime}(F) \\ \mathbf{min} \text{ Until}(\mathbf{Form} F_1, \mathbf{Form} F_2) &= F_2 \vee (F_1 \wedge \bigcirc \text{Until}(F_1, F_2)) \\ \mathbf{min} \text{ Since}(\mathbf{Form} F_1, \mathbf{Form} F_2) &= (F_2 \vee (F_1 \wedge \odot \text{Since}(F_1, F_2))) \end{aligned}$$

The qualifiers **max** and **min** indicate the positive and, respectively, negative interpretation that is to be given to the associated temporal predicate at trace boundaries — corresponding to maximal and minimal solutions to the equations. Thus $\bigcirc \text{Always}(p)$ is defined to be true in the last state of a given trace, whereas $\bigcirc \text{Until}(p, q)$ is false in the last state. Thus the formula $\text{Always}(p)$ will hold on a finite sequence from, say index i , if and only if p holds in every state from index i up to and including the final state. Whereas if $\text{Until}(p, q)$ holds at index i then q must be true at some state with index $j \geq i$ and p true on all states from i up to but not including j .¹

Even without data parametrization, the primitive concatenation temporal operators together with the recursively defined temporal predicates take the logic into the world of context-free expressivity. The classic example of matching parentheses can be captured by $\text{Match}(\textit{begin}, \textit{end})$ where

$$\begin{aligned} \mathbf{min} \text{ Match}(\mathbf{Form} B, \mathbf{Form} E) &= \\ &B \cdot \text{Match}(B, E) \cdot E \cdot \text{Match}(B, E) \vee \text{Empty}() \end{aligned}$$

with $\text{Empty}()$ true just on the empty sequence. Parametrization of temporal predicates by data values allows us to define real-time and stochastic logical operators. To address real-time, for example, we assume that EAGLE is monitoring time-stamped states, where the state contains a variable *clock* holding the associated real time. Then it becomes straightforward to define real-time qualified temporal operators such as *happens before real time u*.

$$\begin{aligned} \mathbf{min} \text{ HappensBefore}(\mathbf{Form} F, \mathbf{double} u) &= \\ &\textit{clock} < u \wedge (F \vee (\neg F \wedge \bigcirc \text{HappensBefore}(F, u))) \end{aligned}$$

It should be clear how more complex real-time and probabilistic temporal operators can be defined.

We still claim that EAGLE presents a natural rule/equation based language for defining, even programming, monitors for complex temporal behavioural patterns. EAGLE is, however, expressively rich and in general this comes with a potentially high computational cost, practically speaking. So one might ask whether EAGLE presents the most appropriate set of primitive temporal operators for runtime monitoring. The non-deterministic concatenation operator, as used above in the matching parentheses example, requires considerable care in use. In order to achieve the expected temporal behaviour pattern, the formulas passed to Match should specify single state sequences. If that is not the case, the concatenation

¹ Arguments for using other interpretations over finite traces have been put forward. However, we have found that this simple interpretation has been good enough for our monitoring purposes

operator may choose an arbitrary cut point, and therefore skip unmatched B s or E s in order to give a positive result. Later but currently unpublished work developed such arguments further and proposed a variety of deterministic versions of temporal concatenation and chop for run-time monitoring, using different forms of cut, e.g. left and right minimal, left and right maximal, etc..

With respect to the computational effectiveness of algorithms for EAGLE trace-checking, in [3] we showed how trace-checking of full EAGLE can be undertaken on a state-by-state basis, even though the logic has the same temporal expressiveness over the past as over the future; basically, our published trace-check algorithm maintains sufficient knowledge about the past in the evolving monitor formulas. Furthermore, we have shown that for restricted subsets, we can achieve close to optimal complexity bounds for monitoring; one such fragment for which we computed complexity results was the LTL (past and future) fragment of EAGLE [4]. However, again considerable care must be taken with the presence of data arguments in temporal predicates, for an explosion in the size of the evolving monitor formula may occur.

What was clear to us at the time was that there were some practically useful and efficiently executable subsets of EAGLE. Despite the pleasing features of EAGLE, we still believed we should continue to search for a powerful and simpler “core” logic, one that is easy and efficient to evaluate for monitoring purposes. To that end, we present in the remainder of this paper a seemingly simpler, lower-level, rule-based system RULER. In section 2 we introduce RULER and a simple evaluation algorithm by example. Section 3 then provides a more formal semantic treatment and indicates how propositional temporal logic (with past and future operators) can be compiled into RULER. In section 4, we then briefly consider RULER with rule parameters and then present brief conclusions and indicate further work in section 5.

2 RULER by example

Informally, a RULER monitoring program comprises a collection of named rules. A rule is formed from a condition part (antecedent) and a body part (consequent). The rule’s condition may be a conjunctive set of literals, whereas the body is a disjunctive set of conjunctive sets of literals, a literal being a positive or negative occurrence of a rule name or an observation name. The idea is that rules can be made active or inactive, and that the body of an active rule defines what rules and observations must hold in the next observation state under the condition part holding in the current observation. As a very simple example, consider the rule named r below in the context of some observation named a .

$$r : \text{---} \Rightarrow a, r$$

The rule has a vacuous condition. The rule’s body is the conjunctive set containing observation a and rule name r . If we assume that r is active at the start of monitoring, r ’s body asserts that the observation a must hold in next observation state and the rule r must be active again, thus effectively asserting that

observation a must hold in all subsequent observation states. If, at some future observation state, a fails to hold, then there will be a conflict between obligations and actuality and, in this simple case, the rule will fail at that particular point. Given the possibility of choice in the body part of a rule, rule failure may or may not mean that the monitoring program, i.e. the rule set, fails on the trace.

Figure 1 outlines a basic algorithm for monitoring a sequence of observation states with a set of named rules. To demonstrate this algorithm, we'll consider

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create an initial set of initial rule activation states
WHILE observations exist DO
  obtain next observation state
  merge observation state with the set of rule activation states
  raise monitoring exception if there's total conflict
  for each of the current merged states,
    apply activated rules to generate a successor set of activation state
  union successor sets to form the new frontier of rule activation states
OD

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Fig. 1. The basic monitoring algorithm

a set of rules below that capture both past time conditions and future time obligations. Let us assume we wish to monitor some temporal behaviour of a system in terms of two observations, a and b . Thus, we arrange for the system to be instrumented to produce an ordered sequence of observation states and that the letters a and b denote particular propositions over an observation state. In effect, we'll treat an observation trace as a sequence of sets of literals².

Example 1. We wish to monitor the constraint that whenever observation a occurs both now and in the immediate previous state then b must occur as a later real observation. We can characterise this by the linear time temporal logic formula $\Box((a \wedge \odot a) \Rightarrow \diamond b)$ where \diamond is the strict eventually in the future temporal operator, or using the EAGLE temporal predicates defined in Section 1 by the monitoring formula $\text{Always}((a \wedge \odot a) \Rightarrow \text{Sometime}(b))$. In RULER the following set of rules characterise the required temporal behaviour

$$\begin{array}{lll}
r_0 : \neg\ominus & r_1 : a \neg\ominus & r_2 : \\
r_3 : a, r_2 \neg\ominus & r_4 : \neg\ominus &
\end{array}$$

assuming, however, that the monitoring algorithm starts with an initial set of rule activation sets as $\{\{r_0, r_1, \neg r_2, r_3\}\}$. Rule r_0 acts as a generator rule; it ensures persistent activity of itself together with r_1 and r_3 , i.e. the three rules are always to be active. The empty rule r_2 is used to represent that the temporal constraint $\odot a$ holds (hence it is initially inactive). The rule r_1 is then a generator for r_2

² We don't allow both x and $\neg x$ to occur in an observation state, for any x

and can be viewed as the temporal rule “if we have a today then tomorrow we have yesterday a ”. Rule r_4 captures the obligation $\diamond b$, either b holds in the next observation state or $\neg b$ holds together with a continued obligation to $\diamond b$.

For the example observation trace in the table below, we see that in step 4, both a and $\odot a$ are true (in the merged state, both a and r_2 are present) and hence rule r_3 yields two possibilities for step 5. The choice with b holding true conflicts with the observation in step 5 and therefore is eliminated. Rule r_4 is thus active and remains activated until step 7 when b is observed to hold.

Step	Obs.	Rule Activations	Merged States
0	{}	$\{\{r_0, r_1, \neg r_2, r_3\}\}$	$\{\{r_0, r_1, \neg r_2, r_3\}\}$
1	$\{a, b\}$	$\{\{r_0, r_1, r_3\}\}$	$\{\{a, b, r_0, r_1, r_3\}\}$
2	$\{\neg a, b\}$	$\{\{r_0, r_1, r_2, r_3\}\}$	$\{\{\neg a, b, r_0, r_1, r_2, r_3\}\}$
3	$\{a, b\}$	$\{\{r_0, r_1, r_3\}\}$	$\{\{a, b, r_0, r_1, r_3\}\}$
4	$\{a, b\}$	$\{\{r_0, r_1, r_2, r_3\}\}$	$\{\{a, b, r_0, r_1, r_2, r_3\}\}$
5	$\{\neg a, \neg b\}$	$\{\{b, r_0, r_1, r_2, r_3\}, \{\neg b, r_0, r_1, r_2, r_3, r_4\}\}$	$\{\{\neg a, \neg b, r_0, r_1, r_2, r_3, r_4\}\}$
6	$\{a, \neg b\}$	$\{\{b, r_0, r_1, r_3\}, \{\neg b, r_0, r_1, r_3, r_4\}\}$	$\{\{a, \neg b, r_0, r_1, r_3, r_4\}\}$
7	$\{\neg a, b\}$	$\{\{b, r_0, r_1, r_2, r_3\}, \{\neg b, r_0, r_1, r_2, r_3, r_4\}\}$	$\{\{\neg a, b, r_0, r_1, r_2, r_3\}\}$
8	$\{\neg a, \neg b\}$	$\{\{r_0, r_1, r_3\}\}$	$\{\{\neg a, \neg b, r_0, r_1, r_3\}\}$

But how do we determine whether any generated temporal existential obligations, such as $\diamond b$, have indeed been satisfied? Essentially, we mark those rules that correspond to such obligations and then, at the end of monitoring, check whether the final merged state set contains states without those marked rules active. If there are no such states, then the given (finite) observation trace fails to satisfy the rule set. If there is at least one of the possible final states not containing such marked rules, the observation trace satisfies the rule set. The approach is exactly that of the minimal and maximal rule interpretations used in EAGLE. In the above, the final set of merged states has just one state that does not contain rule r_4 and hence the observation satisfies the given rule set.

The rule set used in fact contained an optimisation; the choices appearing in rules r_3 and r_4 were made deterministic, either b or $\neg b \wedge \dots$. The determinisation thus reduced the number of possible successor states that are generated at any one time. For example, if the rules r_3 and r_4 had been defined as

$$r_3 : a, r_2 \rightarrow b \mid r_4 \quad r_4 : \neg b \mid r_4$$

the rule activations for step 7 would be $\{\{b, r_0, r_1, r_2, r_3\}, \{r_0, r_1, r_2, r_3, r_4\}\}$, yielding merged states $\{\{\neg a, b, r_0, r_1, r_2, r_3\}, \{\neg a, b, r_0, r_1, r_2, r_3, r_4\}\}$. Then, step 8 would have had $\{\{r_0, r_1, r_3\}, \{b, r_0, r_1, r_3\}, \{r_0, r_1, r_3, r_4\}\}$ for rule activations and $\{\{\neg a, \neg b, r_0, r_1, r_3\}, \{\neg a, \neg b, r_0, r_1, r_3, r_4\}\}$ for its merged states, one of which does not contain the marked (minimal) rule r_4 and so the observation trace, as is to be expected, satisfies the rule set.

2.1 Inhibiting Rule Activation

The informal semantics we’ve used above has rules being activated in the next step if they appear positively in some applied consequent of some currently

applicable rule. In particular, rules that are not mentioned in a consequent of some rule can not be activated by that rule; however, some other rule may indeed activate them. Consider, for example, the contrived (sub)set of rules below.

$$r_0 : \multimap r_2 | r_3 \qquad r_1 : \multimap r_3 | r_4$$

Assume at some stage that r_0 and r_1 are activated in the same step. Rule r_0 therefore generates the partial successor states $\{r_2\}$ and $\{r_3\}$. Rule r_1 will then extend these states to yield the possible (partial) states $\{r_2, r_3\}$, $\{r_2, r_4\}$, $\{r_3\}$ and $\{r_3, r_4\}$. Suppose it was desired that rules r_2 and r_3 were mutually exclusive. It would be necessary to modify the rules as below.

$$r_0 : \multimap r_2, \neg r_3 | r_3, \neg r_2 \qquad r_1 : \multimap r_3 | r_4$$

Assuming again both r_0 and r_1 active, the possible successor activation sets are now $\{r_2, \neg r_3, r_4\}$, $\{\neg r_2, r_3\}$ and $\{\neg r_2, r_3, r_4\}$ — since the potential rule activation set $\{r_2, \neg r_3, r_3, \neg r_2\}$ is clearly inconsistent. The negation of a rule should be interpreted as a forced “non-activation” of the rule.

In the examples above, we indicated how various temporal conditions could be translated into collections of these low level single-shot (or step?) rules. In a certain sense, rule names can be viewed as propositions denoting temporal subformulas. However, it is important to emphasise that a negated rule name does not correspond to the negation of a subformula that the rule name may be viewed as representing. More strictly, one should view a positive occurrence of a rule name as meaning that the rule will be applied and in doing so will generate possible traces that satisfy the associated subformula. A negative occurrence of a rule name (in the rule activation state) simply means that the rule is NOT applied and hence places no constraints on the generation of traces.

In summary, we can use rules to activate other rules (positive appearance of a rule in a consequent), to not inhibit activation (no mention of a rule in a consequent), and to inhibit activation (negative appearance of a rule in a consequent).

3 Propositional RULER trace semantics

We now present a formalization of propositional rule systems and an evaluation semantics over traces of observations.

Preliminary definitions. Let X denote a set of atoms. We then use X^- to denote the set of negated atoms of X , i.e. $\{\tilde{x} \mid x \in X\}$, and let X^\pm denote the set of literals of X , i.e. $\{x, \tilde{x} \mid x \in X\}$. We use the term X -literal to refer to a member of X^\pm . A set of X -literals L is said to be *self-consistent* if and only for any $x \in X$ it is not the case that both $x \in L$ and $\tilde{x} \in L$. Let $L_{\bar{X}}^*$ denote the negative closure of L (with respect to the atoms X), i.e. the set $L \cup \{\tilde{l} \mid l \notin L, l \in X\}$. Given LS_1 and LS_2 as sets of self-consistent sets of literals, the product $LS_1 \times LS_2$ is the set $\{ls_1 \cup ls_2 \mid ls_1 \in LS_1, ls_2 \in LS_2, \text{ and } ls_1 \cup ls_2 \text{ is self-consistent}\}$.

Rule Systems. Given disjoint sets of rule names R and observations O , a *rule* ρ is a pair $\langle C, B \rangle$ where C , the condition part, is a conjunctive set of $(R \cup O)$ -literals, and B , the body part, is a disjunctive set of conjunctive sets of $(R \cup O)$ -literals. A *named rule* is then an association $r : \rho$ where $r \in R$ is a rule name and ρ is a rule. A *rule system* RS is a tuple $\langle R, O, P, I, F \rangle$ where R and O are, respectively, disjoint sets of rule names and observations, and P is a set of disjointly R -named rules over R and O , $I \subseteq R$ is an initial set of active rule names, and $F \subseteq R$ is a set of terminally excluded rule names. A *configuration* γ for a rule system RS is a pair $\langle A, \Theta \rangle$ where A is a consistent set of R -literals, called the activity set, and Θ is a consistent set of O -literals, called the observation state. We also write $A(\gamma)$ to denote the activity set of a configuration γ , similarly $\Theta(\gamma)$ for the observation state.

We next define the interpretation of a set of literals in a configuration. The presence of a positively signed rule name r in the activity set means that the rule ρ associated with r is active. On the other hand, the presence of a negatively signed rule name r , or the absence of r , in the activity set means that the rule ρ associated with r is not active. For observation atoms, however, undefinedness of an O -literal o , i.e. the absence of o from the observation state of the configuration, means that the observation literal o may be either true or false.

Modelling and step relation. Let $RS = \langle R, O, P, I, F \rangle$ be a rule system. A self-consistent set of literals L from RS holds in a configuration γ for RS , which is denoted by $\gamma \models L$, if and only (i) the set of rule name literals mentioned in L are contained in the negative closure of $A(\gamma)$, i.e. $(L - O^\pm) \subseteq A(\gamma)_R^-$, and (ii) observation literals within L are properly contained in the configuration's set of observations $(L - R^\pm) \subseteq \Theta(\gamma)$. An *r - ρ -step relation* $\xrightarrow{r:\rho}$ between configurations is such that $\gamma \xrightarrow{r:\rho} \gamma'$ if and only if (i) $r \in A(\gamma)$, (ii) $\gamma \models C(\rho)$, and (iii) there is a $\theta \in B(\rho)$ such that $A(\gamma') \cup \Theta(\gamma') = \theta$. Then for a set of rule names R , let Γ' be an *R -indexed set of outcome configurations* such that for each $r \in R$, $\gamma \xrightarrow{r:\rho} \Gamma'_r$. We then define the step relation \longrightarrow between configurations such that $\gamma \longrightarrow \gamma'$ if and only if γ' is a consistent pairwise union of an $(A(\gamma) \cap R)$ -indexed set of outcome configurations from γ . Note that an empty union set is treated as being an inconsistent union.

The single step relation for the rule system can now be used to define the notion of an accepting run of a rule system over a given observation trace. This requires matching obligations against actual observations. Positive literals in obligations must appear in an actual observation set, whereas negative literals in obligations are taken to mean that occurrence of the literal's atom must not occur in the actual observation, i.e. a positive literal. Thus, the observation set $\{p, q\}$ matches the obligation set $\{p\}$, but it does not match the obligation set $\{p, \bar{q}\}$ — q must not be present, whereas the observation set $\{p\}$ does match the obligation set $\{p, \bar{q}\}$. We thus have the following.

Matching. An actual set of observation literals X is said to *match* an obligatory set of literals Y if and only if any literal in the set $Y - X$ is a negative literal.

Finally, we can define the language accepted by a rule system.

Language acceptance. An accepting run of a rule system $RS = \langle R, O, P, I, F \rangle$ on an observation trace $\tau = o_1 o_2 \dots o_n$ is a sequence of configurations $\gamma_1 \gamma_2 \dots \gamma_n \gamma_{n+1}$ such that (i) $A(\gamma_1) \in I$, (ii) for all $i \in 1..n$ the actual and obligated observations, o_i and $\Theta(\gamma_i)$ respectively, match and $\langle A(\gamma_i), \Theta(\gamma_i) \cup o_i \rangle \longrightarrow \gamma_{i+1}$, and (iii) $A(\gamma_{n+1}) \cap F = \{\}$. Thus, the language accepted by a rule system RS , $\mathcal{L}(RS)$, is the set of all finite observation traces τ accepted by RS . Furthermore, we say a rule system RS is violated by an observation trace τ if RS has no accepting run on τ , alternatively, $\tau \notin \mathcal{L}(RS)$.

We now claim that the monitoring algorithm of section 2 accepts an observation trace τ for a rule system RS if and only if $\tau \in \mathcal{L}(RS)$. The steps of the algorithm closely reflect the semantic construction we have given.

3.1 Propositional linear temporal logic as a rule system

Having formally defined propositional rule systems, we are now in a position to show how linear-time temporal logic formulas for monitoring over finite traces can be encoded in RULER. Our translation is based on the separation result of Gabbay (originally 1981 but elaborated in [7]), which can then be used to show that any mixed past, present and future linear-time temporal formula can be translated into a collection of universal implications of the form *non-strict past formula* implies *pure future formula*, a minor variation of the rule forms used in the executable temporal logic METATEM [2]. Our starting point is thus to show how such separated temporal implications can be represented in RULER.³

The pure future part. We assume pure future linear time temporal formula built from propositions, the boolean connectives and, or, and negation, \wedge , \vee and \neg , respectively, and a strict until and unless operator, \mathcal{U}^+ and \mathcal{W}^+ . All other standard future time operators are definable from this set. Without loss of generality, we assume formulas are further transformed in negation normal form (NNF), i.e. negation operators pushed inwards to propositional literals and cancellations applied. Let WFF^+ denote the set of well-formed strict future time formulas in NNF and WFF denote the set of well-formed future time formulas in NNF (which may include the present, i.e. propositions under no future time operator).

³ Fisher's SNF representation for temporal logic [5] is close to RULER rule forms and an alternative translation to a rule system could be given via SNF. However, we believe our direct translation has interest in its own right and might lead to an easier SNF translation.

We define a translation $Tr : WFF \rightarrow RuleSystem$ inductively over the structure of the temporal formulas. Let ϕ and ψ denote arbitrary members of WFF . The base cases of the translation are straightforward, e.g. for an atom p , we have $Tr(p) = \langle \{\}, \{p\}, \{\}, \{\{p\}\}, \{\}\rangle$, indicating a rule system with an atom p with an initial set of active rule names containing the singleton set $\{p\}$. Negated atoms translate in a similar way. The propositional constant **true** gives rise to the rule system $\langle \{\}, \{\}, \{\}, \{\{\}\}, \{\}\rangle$ whereas **false** translates to a system with an empty set of initial states. As one might expect, the logical conjunction (disjunction) of formulas ϕ and ψ translate to the obvious product (union) operations that can be defined for rule systems. This leaves the most interesting part of the translation, namely an until formula $\phi\mathcal{U}^+\psi$.

$$\begin{aligned} Tr(\phi\mathcal{U}^+\psi) = & \text{LET } \langle R_\phi, O_\phi, P_\phi, I_\phi, F_\phi \rangle = Tr(\phi) \text{ AND} \\ & \langle R_\psi, O_\psi, P_\psi, I_\psi, F_\psi \rangle = Tr(\psi) \\ \text{IN } & \langle R_\phi \cup R_\psi \cup \{r_{\phi\mathcal{U}^+\psi}\}, \\ & O_\phi \cup O_\psi, \\ & P_\phi \cup P_\psi \cup \{r_{\phi\mathcal{U}^+\psi} : \neg\rightarrow I_\psi \cup (I_\phi \times \{\{r_{\phi\mathcal{U}^+\psi}\}\})\}, \\ & \{\{r_{\phi\mathcal{U}^+\psi}\}\}, \\ & F_\phi \cup F_\psi \cup \{r_{\phi\mathcal{U}^+\psi}\} \rangle \end{aligned}$$

For ease of understanding, we have subscripted the rule names by the subformulas they represent. As the until operator has a strong interpretation, requiring its second argument to be satisfied, the associated rule name for the until formula must be included in the F set of the rule system. As might be expected, the translation of the unless formula differs from the until translation just in the non-inclusion of the rule for the unless formula in the F set.

Example 2. Assume a, b, c and d are atomic propositions. The translation of $a\mathcal{U}^+b$ yields the rule system

$$\langle \{r_{a\mathcal{U}^+b}\}, \{a, b\}, \{r_{a\mathcal{U}^+b} : \neg\rightarrow b \mid a, r_{a\mathcal{U}^+b}\}, \{\{r_{a\mathcal{U}^+b}\}\}, \{r_{a\mathcal{U}^+b}\} \rangle$$

Similarly, the translation of $a \wedge (c\mathcal{W}^+d)$ yields the rule system

$$\langle \{r_{c\mathcal{W}^+d}\}, \{a, c, d\}, \{r_{c\mathcal{W}^+d} : \neg\rightarrow d \mid c, r_{c\mathcal{W}^+d}\}, \{\{a, r_{c\mathcal{W}^+d}\}\}, \{r_{c\mathcal{W}^+d}\} \rangle$$

Thus the translation of $(a\mathcal{U}^+b)\mathcal{U}^+(a \wedge (c\mathcal{W}^+d))$ yields the rule system

$$\langle \{r_0, r_1, r_2\}, \{a, b, c, d\}, \left\{ \begin{array}{l} r_0 : \neg\rightarrow b \mid a, r_0 \\ r_1 : \neg\rightarrow d \mid c, r_1 \\ r_2 : \neg\rightarrow a, r_1 \mid r_0, r_2 \end{array} \right\}, \{\{r_2\}\}, \{r_0, r_2\} \rangle$$

where

$$r_0 = r_{a\mathcal{U}^+b}, \quad r_1 = r_{c\mathcal{W}^+d}, \quad r_2 = r_{(a\mathcal{U}^+b)\mathcal{U}^+(a \wedge (c\mathcal{W}^+d))}$$

Past time temporal queries. The pure past time fragment of linear-time temporal logic is constructed in a mirror fashion to the pure future part, i.e. from propositions, the boolean connectives (\wedge , \vee and \neg), and just the temporal operators \mathcal{S}^- (the strict *since*, false at the beginning of time) and its weak version

\mathcal{Z}^- (true at the beginning of time). As before without loss of generality, we assume that past time temporal formulas have negations pushed inwards to atomic propositions (or literals) and double negations are cancelled. Let us first consider the translation of pure past time temporal queries. The temporal equivalence $\phi\mathcal{S}^-\psi \Leftrightarrow \odot\psi \vee \odot\phi \wedge \odot(\phi\mathcal{S}^-\psi)$ should serve as a reminder of the semantics that needs to be captured by the translation. The basic idea for handling the past is an old one, namely, we use the translation rules to calculate the value of the temporal query as we proceed in time (rather than evaluating the query over the history). We will use the presence of the rule name $r_{\phi\mathcal{S}^-\psi}$ in the rule activation state to denote whether the temporal formula $\phi\mathcal{S}^-\psi$ held in the previous moment (similarly for r_ϕ and r_ψ). We then use a rule, named $r_{\psi:\psi\mathcal{S}^-\phi?}$, to calculate whether $r_{\phi\mathcal{S}^-\psi}$ should be made active because ψ held in the previous moment (similarly for the other possible way for $\psi\mathcal{S}^-\phi$ to hold). These query rules must be universally active in order to determine truth values for the next moment. Thus we use a rule, named say $r_{g,\phi\mathcal{S}^-\psi?}$, to act as a generator (hence the “g” in its name) for a pair of (sets of) rules that determine the truth of $\phi\mathcal{S}^-\psi$ based on the previous values of its subformulas.

$$\begin{aligned} r_{\psi,\phi\mathcal{S}^-\psi?} : r_\psi \multimap r_{\phi\mathcal{S}^-\psi} & \quad r_{\phi,\phi\mathcal{S}^-\psi?} : r_\phi, r_{\phi\mathcal{S}^-\psi} \multimap r_{\phi\mathcal{S}^-\psi} \\ r_{g,\phi\mathcal{S}^-\psi?} : \multimap r_{g,\phi\mathcal{S}^-\psi?}, r_{\phi,\phi\mathcal{S}^-\psi?}, r_{\psi,\phi\mathcal{S}^-\psi?} & \end{aligned}$$

Naturally, our translation must take into account the fact that the subformulas ψ and ϕ may be boolean combinations of pure past time temporal formulas (represented by rule names) and/or literals. Let FFF^- denote the set of pure past temporal formulas and FFF^{-0} the set of present and pure past time temporal formulas. We thus define a translation $Tq : FFF^{-0} \rightarrow RuleSystem$ that will translate a past time temporal formula into a rule system whose initial activation set, as a disjunctive set of conjunctive sets of rule names and/or literals, is to be viewed as representing the given temporal query. We assume that ϕ and ψ are arbitrary members of FFF^{-0} . However, we use the F set of the rule system to represent the initial values of rules, e.g. a formula $\phi\mathcal{S}^-\psi$ must be false initially and so the rule name $\neg r_{\phi\mathcal{S}^-\psi}$ would be included in the set F . As with the future time translation the base cases are clear, as is conjunction and disjunction. Figure 2 shows the translation for the interesting case of the strict since operator.

Separated temporal implicative forms. We can now bring together the above two translations Tr and Tq to generate a rule system corresponding to the METATEM-like rule form $\phi_{\text{past}} \Rightarrow \odot\psi_{\text{future}}$ which are of universal nature, i.e. globally hold. Assuming both ϕ_{past} and ψ_{future} are in a negation normal form, then, in the context of

$$\begin{aligned} \langle R_{\text{past}}, O_{\text{past}}, P_{\text{past}}, I_{\text{past}}, F_{\text{past}} \rangle &= Tq(\phi_{\text{past}}) \\ \langle R_{\text{future}}, O_{\text{future}}, P_{\text{future}}, I_{\text{future}}, F_{\text{future}} \rangle &= Tr(\psi_{\text{future}}) \end{aligned}$$

in which we assume without loss of generality the rule name sets are disjoint, the rule system below will represent the translation of the separated implicative form, i.e. $Ti(\phi_{\text{past}} \Rightarrow \odot\psi_{\text{future}})$.

$$\begin{aligned}
Tq(\phi\mathcal{S}^-\psi) = & \\
& \text{LET } \langle R_\phi, O_\phi, P_\phi, I_\phi, F_\phi \rangle = Tq(\phi) \text{ AND} \\
& \langle R_\psi, O_\psi, P_\psi, I_\psi, F_\psi \rangle = Tq(\psi) \\
& \text{IN } \langle R_\phi \cup R_\psi \cup \{r_{\phi\mathcal{S}^-\psi}, r_{g.\phi\mathcal{S}^-\psi?}\} \cup \{r_{\phi\mathcal{S}^-\psi?x} \mid x \in I_\phi\} \cup \{r_{\phi\mathcal{S}^-\psi?x} \mid x \in I_\psi\}, \\
& O_\phi \cup O_\psi, \\
& P_\phi \cup P_\psi \cup \\
& \{r_{g.\phi\mathcal{S}^-\psi?} : \dashv\!\!\dashv \rightarrow \{r_{g.\phi\mathcal{S}^-\psi?}\} \cup \{r_{\phi\mathcal{S}^-\psi?x} \mid x \in I_\phi\} \cup \{r_{\phi\mathcal{S}^-\psi?x} \mid x \in I_\psi\}\} \cup \\
& \{r_{\phi\mathcal{S}^-\psi?x} : x \dashv\!\!\dashv \rightarrow r_{\phi\mathcal{S}^-\psi} \mid x \in I_\psi\} \cup \\
& \{r_{\phi\mathcal{S}^-\psi?x} : x, r_{\phi\mathcal{S}^-\psi} \dashv\!\!\dashv \rightarrow r_{\phi\mathcal{S}^-\psi} \mid x \in I_\phi\}, \\
& \{\{r_{\phi\mathcal{S}^-\psi}, r_{g.\phi\mathcal{S}^-\psi?}\}\}, \\
& F_\phi \cup F_\psi \cup \{\neg r_{\phi\mathcal{S}^-\psi}\} \rangle
\end{aligned}$$

Fig. 2. Translation of $\phi\mathcal{S}^-\psi$

$$\begin{aligned}
RS = & \langle R_{\text{past}} \cup R_{\text{future}} \cup \{r_{g.\phi_{\text{past}} \Rightarrow \psi_{\text{future}}}\} \cup \{r_{x \Rightarrow \psi_{\text{future}}} \mid x \in I_{\text{past}}\}, \\
& O_{\text{past}} \cup O_{\text{future}}, \\
& P_{\text{past}} \cup P_{\text{future}} \cup \\
& \{r_{g.\phi_{\text{past}} \Rightarrow \psi_{\text{future}}} : \dashv\!\!\dashv \rightarrow \{r_{g.\phi_{\text{past}} \Rightarrow \psi_{\text{future}}}\} \cup \{r_{x \Rightarrow \psi_{\text{future}}} \mid x \in I_{\text{past}}\}\}, \\
& \{r_{x \Rightarrow \psi_{\text{future}}} : x \dashv\!\!\dashv \rightarrow I_{\text{future}} \mid x \in I_{\text{past}}\}, \\
& \{\{r_{g.\phi_{\text{past}} \Rightarrow \psi_{\text{future}}}, r_{x \Rightarrow \psi_{\text{future}}} \mid x \in I_{\text{past}}\} \cup F_{\text{past}}\}, \\
& F_{\text{future}} \rangle
\end{aligned}$$

Example 3. Assuming a, b, c, p and q denote propositions, we give the RULER translation of the universal separated temporal implication

$$c \wedge (b\mathcal{S}^-a) \Rightarrow \bigcirc(\diamond p \wedge \diamond q).$$

Recall that $\diamond p$ will be translated as $p \vee \mathbf{true}\mathcal{U}^+p$, i.e. $p \vee \mathbf{true}\mathcal{U}^+p$, similarly for $\diamond q$. Using the following abbreviations

$$\begin{aligned}
r_0 &= r_{g.b\mathcal{S}^-a?} & r_1 &= r_{b\mathcal{S}^-a?b} & r_2 &= r_{b\mathcal{S}^-a?a} & r_3 &= r_{b\mathcal{S}^-a} \\
r_4 &= r_{\mathbf{true}\mathcal{U}^+p} & r_5 &= r_{\mathbf{true}\mathcal{U}^+q} & r_6 &= r_{g.c \wedge (b\mathcal{S}^-a) \Rightarrow \bigcirc((p \vee \mathbf{true}\mathcal{U}^+p) \wedge (q \vee \mathbf{true}\mathcal{U}^+q))} \\
r_7 &= r_{c \wedge (b\mathcal{S}^-a) \Rightarrow \bigcirc((p \vee \mathbf{true}\mathcal{U}^+p) \wedge (q \vee \mathbf{true}\mathcal{U}^+q))}
\end{aligned}$$

the rule system will thus have rules

$$\begin{array}{lll}
r_0 : \dashv\!\!\dashv \rightarrow r_0, r_1, r_2 & r_1 : b, r_3 \dashv\!\!\dashv \rightarrow r_3 & r_2 : a \dashv\!\!\dashv \rightarrow r_3 \\
r_3 : & r_4 : \dashv\!\!\dashv \rightarrow p \mid r_4 & r_5 : \dashv\!\!\dashv \rightarrow q \mid r_5 \\
r_6 : \dashv\!\!\dashv \rightarrow r_6, r_7 & r_7 : c, r_3 \dashv\!\!\dashv \rightarrow p, q \mid p, r_5 \mid r_4, q \mid r_4, r_5
\end{array}$$

with an initial rule activation set as $\{\{-r_3, r_0, r_1, r_2, r_6, r_7\}\}$ and the forbidden rule set as $\{r_4, r_5\}$.

The correctness of our translation scheme for propositional LTL over finite traces with respect to the given semantics for RULER follows from the correctness of separation, then an inductive proof establishing the correctness of the translation of the universal separated implicative temporal forms.

4 Parameterized RULER

The propositional RULER system corresponds to regular-based languages, which are a subclass of propositional EAGLE. Here, we extend RULER to include rule definitions parameterized by rules. The evaluation strategy used on this seemingly small extension increases the formal expressivity of RULER to be beyond context-free languages. Consider the following rule definition, indeed schema, that has been extended to include formal rule arguments.

$$r(\rho) : a \multimap b, \rho \mid c, r(\rho)$$

Suppose that the rule r is active with the propositional rule r_0 substituted for ρ , i.e. $r(r_0)$ is active. Informally, the evaluation of $r(r_0)$ will first determine the truth of the condition part a , then, assuming it holds, continue to create a set of activation states for the next step corresponding to $\{\{b, r_0\}, \{c, r(r_0)\}\}$. Let us give a few examples that show how the expressivity of rule parameterized RULER jumps into the context sensitive languages.

Example 4. Consider a rule system with two rules

$$r_b(\rho) : \multimap b, \neg a, \rho \quad r_{ab}(\rho) : \multimap b, \neg a, \rho \mid a, \neg b, r_{ab}(r_b(\rho))$$

together with an initial rule activation set as $\{\{a, r_{ab}()\}\}$ and the final rule activation set $\{r_{ab}\}$ (meaning that no occurrence of rule r_{ab} may appear as an obligation in a final rule activation state). All accepted observation traces will match against a trace of n occurrences of a followed by n occurrences of b . Essentially, barring the first a , the rule r_{ab} represents the non-terminal S of the context free grammar $S = ab \mid aSb$ in which r_{ab} 's actual argument represents the continuation string for concatenation to the string of a 's generated. It is thus relatively easy to establish that the class of context free languages are a subset of parameterized RULER. We extend the above example to represent traces of the form $a^n b^n c^m$, for $n, m \geq 1$. Take the rule set

$$\begin{aligned} r_b(\rho) &: \multimap b, \neg a, \neg c, \rho \\ r_{ab}(\rho) &: \multimap b, \neg a, \neg c, \rho \mid a, \neg b, \neg c, r_{ab}(r_b(\rho)) \\ r_c &: \multimap c \neg a, \neg b \mid c \neg a, \neg b, r_c \end{aligned}$$

together with an initial activation set as $\{\{a, r_{ab}(r_c)\}\}$ and the final rule activation set $\{r_{ab}, r_c\}$. This system will clearly accept traces of the form $a^n b^n$ (represented by the r_{ab} rule) followed by one or more c 's (determined by the r_c argument to the initial rule activation r_{ab}). Now we can encode the intersection of the languages $a^n b^n c^m$ and $a^m b^n c^n$ ($n, m \geq 1$), thus yielding the context sensitive language containing words of the form $a^n b^n c^n$.

$$\begin{aligned} r_b(\rho) &: \multimap b, \neg a, \neg c, \rho & r_a(\rho) &: \multimap a, \rho \mid a, r_a(\rho) \\ r_{ab}(\rho) &: \multimap b, \neg a, \neg c, \rho \mid a, \neg b, \neg c, r_{ab}(r_b(\rho)) & r_{bc}(\rho) &: \multimap c, \neg a, \neg b, \rho \mid b, \neg a, \neg c, r_{bc}(r_{c1}(\rho)) \\ r_c &: \multimap c \neg a, \neg b \mid c \neg a, \neg b, r_c & r_{c1}(\rho) &: \multimap c, \neg a, \neg b, \rho \end{aligned}$$

where the rule system then has an initial rule activation set $\{a, r_{ab}(r_c), r_a(r_{bc}())\}$ and forbidden final rule activation set $\{r_{ab}, r_c, r_a, r_{bc}\}$.

5 Conclusions

We have introduced a low-level rule system RULER as a kind of “byte-code” for run-time monitoring logics. A basic monitoring algorithm was described for the propositional subset of RULER. Having presented formally the semantics of the propositional subset, we demonstrated how linear time temporal logic with both past and future operators can translate to such rule systems, and then briefly, and informally, presented RULER where rules are parameterized by rule names. On the face of it, the propositional subset of RULER looks rather like a grammatical representation of the transition relation of an alternating automaton, i.e. with conjunctive and disjunctive branching, see for example [6]. However, RULER, even the propositional subset, has more to it; the rules have the capability to switch other rules on or off as an evaluation of a rule system over a trace proceeds. We are referring to such systems as reactive rule systems / grammars / Kripke structures [8]. Whilst regular grammars are closed under this notion of reactivity, it can easily be shown that reactive context free grammars take us beyond context free. Some relationship with state-alternating context-free [9] is clear, however, a more detailed study of reactive grammars and their place in the complexity hierarchy is work in progress. A feature we haven’t yet mentioned is rule priority in RULER. Given the ability to switch rules on and off, conflicts may occur. Sometimes the conflicts may be desired, but in other situations we may wish one rule to override another, as is the case in defeasible reasoning. Of course, this changes the nature of the logics expressible quite considerably. In addition to rule parameters, RULER has data parameters, just as in EAGLE. The semantic details are not difficult and we adopt an approach similar to that in first-order METATEM [1], but, just as in EAGLE, some care needs to be taken to avoid “rule activation state set” explosion in practice.

A prototype Java implementation of the monitoring algorithm for propositional RULER has been developed, as a proof of concept. We are, however, not yet at the stage where we can properly evaluate the practical effectiveness of RULER, which requires the fully parameterized version of RULER. We hope to report on this in the near future.

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